Determination of Natural Frequency of a Turning Specimen Subjected to Random Excitation

M.Z. Hussain[1], Dr. A. A. Khan[2] and **Dr. M. Suhaib[3]**

[1,3] Department of Mechanical Engineering, Jamia Millia Islamia, New Delhi 110025 India
[2]Department of Mechanical Engineering, Aligarh Muslim University, Aligarh 202002 India
[1]zafer.amu@gmail.com, [2]aalikhan.me@amu.ac.in, [3]msuhaib@jmi.ac.in

Abstract -- **This paper deals with determination of natural frequency of a turning specimen subjected to random cutting and thrust force. In this paper, a machine tool system is modeled as a cross-coupled linear system. The natural frequency of a turning specimen subjected to random excitation can be determined by using Volterra series. The Volterra series represents the response of a system in a functional form, through a series of first and higher order convolution integrals, involving explicit operations on the input to the system. Exploring the input identification procedure for the estimation of excitation forces from the knowledge of system parameters and response of linear cross-coupled system having cross-coupling in stiffness as well as damping. Modeling is done for a machine turning tool system which is being excited by random forces. The turning specimen can be modeled as a two-degree-of freedom system with both direct as well as cross coupling effect has to be considered in linear stiffness and damping terms. The equation of motion has been written in a non-dimensional form. Illustration of procedure is done through numerical simulation. The assumption involved and the approximations are also discussed. The procedure for identification of response and consecutively natural frequency is illustrated through numerical simulation using FORTRAN language.**

Keywords: *Turning Specimen, Machine Tool System, Random Excitation, Volterra Series, Cross Coupling Effect.*

I. INTRODUCTION

THE AIM of this paper is to develop an algorithm that can be used to determine the natural frequency by knowing the response and the system parameter using Volterra series. Such analysis have attracted considerable attention in the recent past, partly due to growing awareness of the significance of the Random nature of forces produced by during operation. Indirect estimation of excitation force using model co-ordinate transformation has been carried out by De Sanghere et.al. [1], classification of different force identification problem has been carried out by Stevens [2]. A study for finding inverse method for estimation of impulsive loads has been conducted by Ma et.al. [3]. A non-linear vibration problem of estimating the external forces for a single degree of freedom system using conjugate gradient method has been developed by Huang [4-5].

Chatter in turning or cutting is usually assumed to be a regenerative process (there exist also other explanations like thermoplastic changes in the material). Because of external disturbance, work piece starts an oscillation relative to the tool, producing a wavy surface. Therefore, the chip thickness that has to be cut in the next round will also vary randomly. Since random cutting force depends on chip thickness fluctuation. In processes like cutting and turning the vibrations that produce the random cutting force are orthogonal to the rotation. But there would be a random input force in the direction of rotation to generate chatter. In this work the input random force is assumed to have white noise type of power spectral density. Such white noise analysis is considered to be an effective tool for gaining a maximum of information with a minimum number of assumptions about the system.

In the present, analysis a two-degree of freedom turning tool system has been modeled having cross-coupled linear damping and stiffness parameters. The equations of motion have been derived from the configuration of the system. These equations are then non-dimensionalised. Laplace transforms are employed to derive expressions for the first order direct and cross-Kernel transforms from the non-dimensionalised equation of motion. These first order direct and cross-Kernels are obtained in the frequency domain, from the knowledge of the system parameters, which are then used to estimate the excitation force.

II. LITERATURE REVIEW

The forward analysis of response generation of a multi degree of freedom system subjected to random excitation force has been the major thrust area in the recent past. The complete inverse analysis of determination of the forcing function is a complex phenomenon and requires specialized techniques particularly when the excitation is random in nature.

The methods that are being used nowadays either involves a detailed knowledge of the material characteristics of the test structures (finite element model approach) Dobling & Farrar [7] or make very restrictive assumption about the excitation Randall & Swevers [8]. Recently, a novel approach was presented for the normalization of operational mode shapes on a basis of in operational model models only Parloo & Guillaume [9]. In this method it is shown that the operational mode shapes can be normalized by means of the measured shift in natural frequencies between the original and mass loaded condition just by adding or removing, for instance one (or more) masses (with well known weights) to the test structures. A complete modal model can be reconstructed by normalizing the operational mode shapes. Using this model, an inverse problem can be formulated for the identification of the unknown forces that gives rise to the measured responses. In reference Guillaume, Verboven & Sitter [10] the forced identification problem was addressed and an inverse solver was proposed and is then compared to classical approach (e.g. pseudo-inverse). For force identification on the basis of output only data an experimental work has been carried out by E Parloo et al. [13].

In this paper the inverse problem of input identification is based on the Volterra approaches, and is discussed in the context of machine tool system. Problem involving vibration arises quite frequently in turning process, especially those involving random vibration of sliding contact element of machine tool system due to varying chip thickness. Modeling of turning process has been done by Wang and Su [17] and they developed the relationship between the cutting force and chip thickness fluctuation will be treated as a hysteresis model. Wang and Su [17] have modeled the turning process as a two degree of freedom system.

The Volterra kernels extraction is proposed for the estimation of fourteen-linearised machine tool parameter subjected to random cutting force by Khan and Khan [12]. The procedure developed gives very good engineering estimates of the machine tool parameters. It can be shown that the large sample size is preferable for obtaining sharp peaks which gives more accurate results. It has been also found that the procedure developed is robust to the measurement noise and can be successfully employed for modeling of the setup.

III. MACHINE TOOL SYSTEM

Problems involving vibration arise quite frequently in machine tool application, especially those involving random vibrations of cutting tools and tool chatter, excited by random cutting force. In some cases, deterministic models prove to be inadequate or at least extremely complex and the phenomenon can be adequately described only within the framework of statistical models. Statistical dynamics concerned with the study of various random phenomena in dynamic systems enriches the classical basic theory of oscillations and extends the possibilities for its applications to the description and analysis of real response processes in dynamic systems. Inverse problems in vibration analysis require techniques with rigorous theoretical base, which provide valid routes to input identification. Further the estimation procedure becomes more complex if the excitation is random in nature. The Volterra [6] series provides a basis for these requirements. The basic theory of Volterra series involves modeling the relationship between the system response and input in terms of a series of first and higher order convolution integrals. It employs multi-dimensional kernels, which upon convolution with the applied excitation express the response in the form of a power series. The kernels of the system are considered as multi-dimensional unit impulse response functions.

IV. GOVERNING EQUATION OF MOTION

A cutting tool used for plain turning operation can be represented as shown in Fig.4.1, where the flexibility is represented by eight springs & damping coefficients is discussed earlier. The excitation forces are denoted as the thrust and main cutting forces exciting the structure.

The equations of motion in the vertical and horizontal direction relating the displacement to the forces applied to it is given by

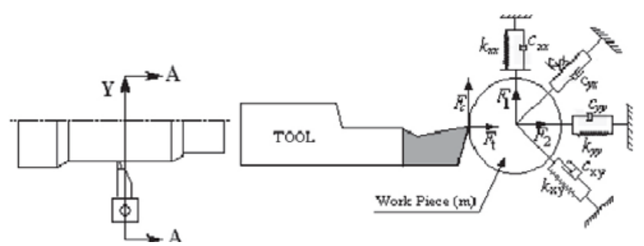

Figure 1. A plain turning tool system with cross-couplings.

$$m\ddot{x}+c_{xx}\dot{x}+c_{xy}\dot{y}+k_{xx}x+k_{xy}y=F_1(t) \quad (1)$$

$$m\ddot{y}+c_{yx}\dot{x}+c_{yy}\dot{y}+k_{yx}x+k_{yy}y=F_2(t) \quad (2)$$

where m is the mass, c_{xx}, c_{yy} are direct linear damping terms; are cross-coupled linear damping coefficients and are the direct linear stiffness terms, while are the cross-coupled linear stiffness term. , represents the excitation forces given to the system in x and y direction in the above equation

In order to write the equation of motion in a non-dimensional form, let us define

$$p=\sqrt{\frac{k_{xx}}{m}}, \quad \xi_{ii}=\frac{c_{ii}p}{2mp^2}, \quad {}^x\eta=\frac{x}{X_{st}}, \quad \tau=pt,$$

$${}^y\eta=\frac{y}{X_{st}}, \quad \varepsilon_{ij}=\frac{k_{ij}}{k_{xx}}, \quad \overline{f_i}(\tau)=\frac{f_i(\tau)}{F_{\max}}, \quad X_{st}=\frac{F_{\max}}{k_{xx}}$$

Where $i=x, y$; $j=x, y$

The non-dimensional form of equation (4.1) and (4.2) becomes

$${}^x\eta''+2\xi_{xx}{}^x\eta'+2\xi_{xy}{}^y\eta'+{}^x\eta+\varepsilon_{xy}{}^y\eta=\overline{f_1}(\tau) \quad (3)$$

$${}^y\eta''+2\xi_{yy}{}^y\eta'+2\xi_{yx}{}^x\eta'+\varepsilon_{yy}{}^y\eta+\varepsilon_{yx}{}^x\eta=\overline{f_2}(\tau) \quad (4)$$

Where ($'$) denotes differentiation with respect to τ .

Taking Laplace transform of equation (4.3) & (4.4) we get

$$s^2\, {}^x\eta(s) + 2s\xi_{xx}\, {}^x\eta(s) + 2s\xi_{xy}\, {}^y\eta(s) + {}^x\eta(s) + \varepsilon_{xy}\, {}^y\eta(s) = F_1(s) \quad (5)$$

$$s^2\, {}^y\eta(s) + 2s\xi_{yy}\, {}^y\eta(s) + 2s\xi_{yx}\, {}^x\eta(s) + \varepsilon_{yy}\, {}^y\eta(s) + \varepsilon_{yx}\, {}^x\eta(s) = F_2(s) \quad (6)$$

Solving the two simultaneous equations (4.5) and (4.6) we get

$${}^x\eta(s) = \frac{(s^2 + 2\xi_{yy}s + \varepsilon_{yy})F_1(s) - (2\xi_{xy}s + \varepsilon_{xy})F_2(s)}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \varepsilon_{yy}) - (2\xi_{yx}s + \varepsilon_{yx})(2\xi_{xy}s + \varepsilon_{xy})} \quad (7)$$

and

$${}^y\eta(s) = \frac{(s^2 + 2\xi_{xx}s + 1)F_2(s) - (2\xi_{yx}s + \varepsilon_{yx})F_1(s)}{(s^2 + 2\xi_{xx}s + 1)(s^2 + 2\xi_{yy}s + \varepsilon_{yy}) - (2\xi_{yx}s + \varepsilon_{yx})(2\xi_{xy}s + \varepsilon_{xy})} \quad (8)$$

V. COMPUTER SIMULATION

The input identification procedure is illustrated through numerical simulation of the response for the non-dimensional coupled equation (3) and (4). The forcing functions chosen for response simulation are normalized zero mean random forces, and . The excitation forces are simulated through random number generating subroutines and are normalized with respect to their maximum values. The governing equations are then numerically solved using a fourth order Runge-Kutta method, to obtain the responses in and directions (and) from equations (3) and (4). Using FFT the power spectrum of the response averaged over the ensemble of 2000 samples is determined.

0.094358

Owing to the statistical nature of the problem, the procedure is illustrated for various sets of direct and coupled stiffness and damping parameters. The case studied for a particular set of stiffness as well as damping parameter have been designed to find the natural frequency of the system using the algorithm developed.

VI. RESULTS AND DISCUSSION

Case I:

$$\varepsilon_{xx} = 2.00,\ \varepsilon_{xy} = 1.00,\ \varepsilon_{yx} = 1.00,\ \varepsilon_{yy} = 2.00$$
$$\xi_{xx} = 0.10,\ \xi_{xy} = 0.01,\ \xi_{yx} = 0.01,\ \xi_{yy} = 0.10$$

For the above set of values of the parameters, the response is computationally simulated using equations (3) and (4). The governing equations are then numerically solved using a fourth order Runge-Kutta method, to obtain the responses in and directions (and). Their corresponding power spectrums averaged over 2000 samples are shown in Fig. 5 and 6. From the power spectrum of the responses the fundamental frequencies of the system are found to be as per unit of non-dimensional frequency ω_2=0.256031 and per unit of non-dimensional frequency these responses are fed as inputs to the input identification algorithm. The various direct and cross-coupled first order kernels are obtained from the knowledge of the system parameters. Typical sample of excitation force used for simulation is shown in Fig. 2. Volterra kernel transforms exhibit the two fundamental frequencies of the system, as it is a two-degree of freedom system. The excitation force has been estimated using these kernels and response. Figure 3 shows the Power spectrum of input force. Figure 4 shows the FFT of the simulated excitation force averaged over the ensemble of 2000 samples and is used for both x and y directions for the case. The corresponding estimated force can be seen in Fig 4.11 and can be compared with the simulated force as shown in Fig. 4. The error in the estimate that is the difference between the Fig.4 (simulated force) and Fig. 7 (estimated force) can be seen in Fig. 8.

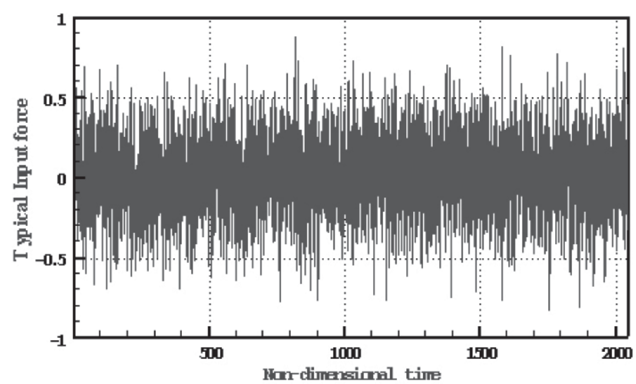

Figure 2. Typical sample of normalized input force.

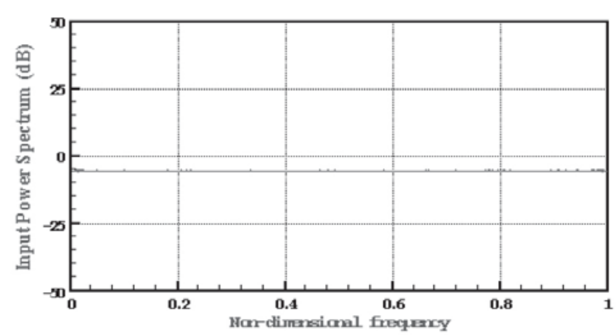

Figure 3. Power spectrum of input force.

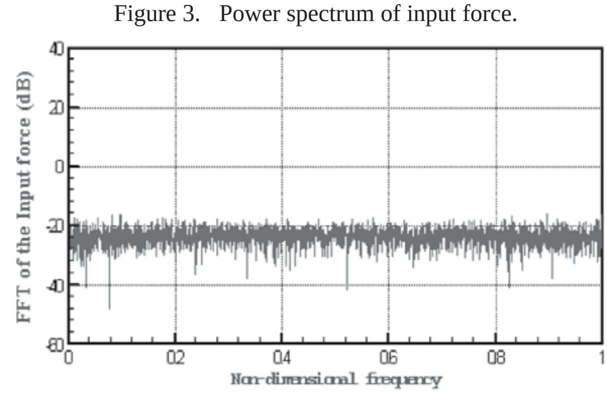

Figure 4. Fast Fourier Transform of the simulated input .

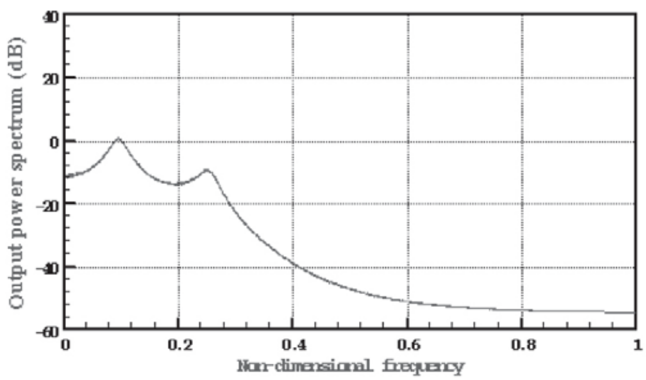

Figure 5. Power spectrum of the response : Case 1.

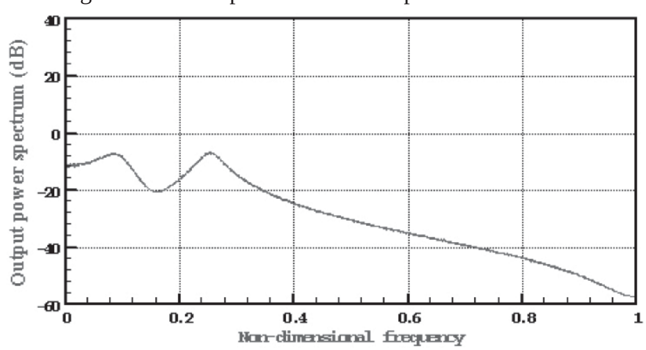

Figure 6. Power spectrum of the response : Case 1.

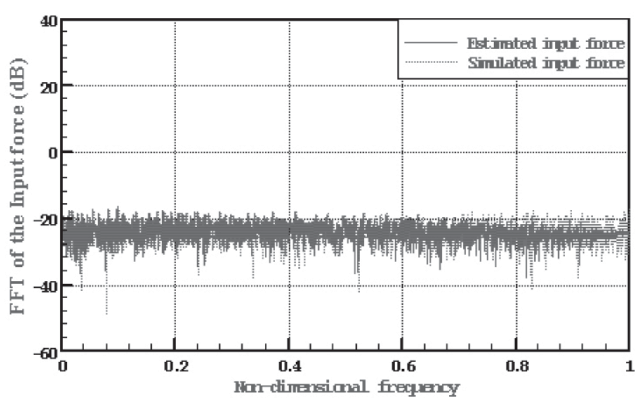

Figure 7. Estimate of the input force $F_1(\omega)$: Case I.

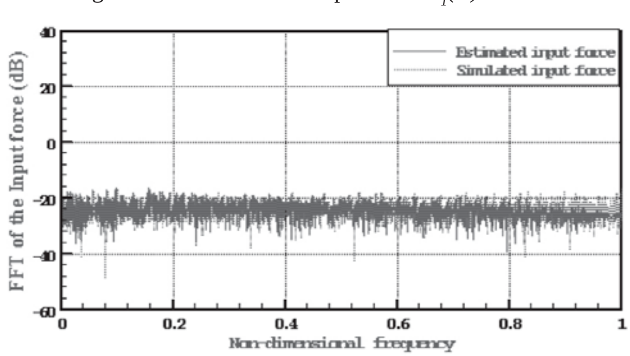

Figure 8. Estimate of the input force : Case I.

VII. CONCLUSION AND SCOPE FOR FUTURE WORK

The present work is primarily concerned with the determination of two fundamental frequencies of cross-coupled turning tool systems. Volterra theories have been employed for the analysis of the problem. A frequency domain approach has been adopted in order to reduce the computation time. The coupled systems considered are machine tool system. The work piece mounted on chuck with cross coupling in damping and stiffness has been considered.

The input identification procedure in cross-coupled system is developed in steps. This serves to illustrate general nature of the procedures adopted for identification of excitation force, which are random in nature. Few case studies have been carried out with different sets of non-dimensional parameters in order to check the accuracy of estimated forcing function. The accuracy of the estimates with various other non-dimensional parameters and level of excitation can be expected to follow the same trends as discussed. The response of the system is expressed through first order direct and cross Volterra kernels. These Volterra kernels are then processed to estimate the excitation force. Reasonably good estimates are found for different sets of both linear damping and stiffness parameters.

The excitation force are found to be estimated with a good degree of accuracy in given case, if the programme run for other cases also however the accuracy of the estimates are found to vary with the values of non-dimensional parameter chosen for simulation. The present study can be used to design experiments to choose appropriate set of excitation levels for the expected sets of parameters. This work can be extended to include the kernels of higher order to increase the accuracy of estimates. Further we are considering stiffness and damping to be linear in order to keep the algebra simple, however an extension to the present work can be made by considering non-linearity in damping and stiffness.

In the present work we have considered the tool to be rigid. Future work needs to be carried out by taking into consideration the flexibility of the tool. The effect of ensemble size is also studied. It has been found that accuracy of the estimates increases with the ensemble size. In the present study the ensemble size of the force and the response is kept 2000, to limit the computation time. Future work can be carried out by increasing the ensemble size in order to increase the accuracy as well as frequency range of force identification.

The analytical development in the present work needs to be validated through experiments in laboratory as a future work. Further experimental work needs to be carried out on turning tool system, milling cutters, drilling tool as well as broaching tool.

VIII. REFERENCES

[1]. G. Desanghere and R. Snoeys, "Indirect Identification of excitation of Excitation force by Modal Co-Ordinate Transformation" Proceedings of the international modal analysis conference, Orlando, FL, 1985 , pp. 685-690.

[2]. K.K. Stevens, "Force Identification Problems-an Overview", *Proc. SEM Spring Conference on Experimental Mechanics*, Houston. 1987, pp.838-844.

[3]. C.K. Ma, P.C. Tuan, D.C. Lin and C.S. Liu, "A study of an inverse method for impulsive load estimation," *International Journal of System Science*, Vol. 29, 1985, pp. 663-672.

[4]. C.H. Huang, "An inverse non linear force vibration problem of estimating the external forces in a damped system in time dependent system parameters,"*Journal of Sound and Vibration* 242, 2001, pp 749-756.

[5]. C.H. Huang, "A Nonlinear Inverse problem of estimating the external forces for a system with displacement dependent parameters", *Journal of Sound and Vibration*, 248, No. 5, 2001, pp. 789-807.

[6]. V. Volterra, *Theory of functional and of Integral and Integro – Differential Equations*, Dover publications, Inc., New York, 1959.

[7]. S.W. Docbling. C.R. Farrar, "Computational of structural flexibility for bridge health monitoring using ambient modal data" *Proc. 11th ASCE engineering mechanics conference*.1996. pp. 1114-1117.

[8]. R.B. Randall, Y. Gao, J. Swevers, Updating modal models from response measurements, *Proc. 33rd International Conference on Noise and Vibration Engineering* (ISMA23) 1998, pp. 1153-1160.

[9]. E. Parloo, P. Verboven, P. Guillaume, M. Van Overmeire "Sensitivity-based mass- normalization of mode shape estimates from output-only data", *Proc. International Conference On Structural System Identification*, Kassel, Germany, 2001, pp. 627-636.

[10]. P.Guillaume, E.Parloo, P.Verboven and G.DeSitter, "An inverse method for the identification of localized excitation sources", *Proc. 20th International Modal Analysis Conference* (IMAC20), 2002.

[11]. Jian Wang and Chun-Yi Su, Two Degree-of-Freedom Modeling and Robust Chatter Control in Metal Cutting by Piezoelectricity, *Proc. 42nd IEEE Conference on Decision and Control* Maui, Hawaii USA, December 2003.

[12]. A.A Khan and A.H. Khan, "Identification of machine tool parameters under random cutting force", *Proc. AIME-2006*, Jamia, New Delhi pp75-85.

[13]. E. Parloo P. Verboven P. Guillaume and M. Van Overmeire "Sensitivity-based damage assessment technique for output-only data", *Proc. Third International Workshop on Structural Health Monitoring*, Sanford, CA, 2001.

Md. Zafer Hussain received the B. Tech. degree in Mechanical Engineering from Aligarh Muslim University (AMU) Aligarh, Aligarh, Uttar Pradesh in 2004, and the M. Tech. degree in Mechanical Engineering with specialization in Machine Design from AMU Aligarh, Aligarh, Uttar Pradesh in 2007, and Pursuing Ph.D. in Mechanical Engineering from Jamia Millia Islamia (A Central University), New Delhi, respectively. Currently, he is a research scholar of Mechanical Engineering at Jamia Millia Islamia, New Delhi. His teaching and research areas include Vibration, Robotics, Mechatronics, Mechanism & Automation, Nonlinear Dynamics, System Identification, etc. He is currently writing a Textbook and authored many research papers.

Dr. Ahmad Ali Khan received the B.S. degree in Mechanical Engineering from Aligarh Muslim University, Aligarh, Uttar Pradesh in 1987, the M.S. degree in Mechanical Engineering from AMU Aligarh, Aligarh, Uttar Pradesh in 1992, and the Ph.D. degree in Mechanical Engineering from Indian Institute the Technology, Kanpur UP in 2000 respectively.

Currently, he is Professor of Mechanical Engineering at AMU Aligarh. His teaching and research areas include Nonlinear Dynamics, System Identification, Smart Structures, etc. He has authored/co-authored many research papers.

Dr. M. Suhaib received the B.Sc. Engg degree in Mechanical Engineering from the Aligarh Muslim University, Aligarh, U.P, in 1990, the M.Sc. Engineering degree in Machine Design from the Aligarh Muslim University, Aligarh, U.P, in 1993, and the Ph.D. degree in Mechanical Engineering from the Jamia Millia Islamia, New Delhi, 2004, respectively. Currently, he is a Professor of Mechanical Engineering at Jamia Millia Islamia. His teaching and research areas include Robotics, Mechatronics, Mechanism and Automation. He has authored/co-authored two textbooks & approximately fifty research papers.