Relationship between Mathematics and Physics: A Fresh Perspective

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Abstract -- **The connection between mathematics and physics is as old as the history of science. With the progress in time the relationship has only deepened. Without exaggeration it can be said that advent of each new revolution in natural science has brought into picture another profound link between the two sciences. In this article, a connection between the two has been looked from a fresh perspective. The emphasis is on delineating the mathematical thinking from the modern physical way of doing things, rather than to find a connection. In this endeavor, one standard problem of one of the two streams is picked up and has been tried to resolve by a different and more mathematical (or physical) method as the case may be. This is done to highlight the basic difference between physics and mathematics.**

Keywords: Mathematics, Physics, Difference between Mathematics and Physics

I. INTRODUCTION

IF ONE looks at the history of natural sciences one can easily find different instances of physics and mathematics going hand in hand. During the new age in western thinking at the time of the European renaissance it was Galileo who first made a connection between the Cartesian geometry and basic physical constructs such as position and velocity. Newton in his "Mathematical Principles of Natural Philosophy" gave us the differential and integral calculus which found immense application in the field of physics. Einstein found links between geometry of space – time and gravitation. These three examples each herald a new revolution in the scientific field.

It is therefore a curiosity to quote Nobel laureate P.A.M. Dirac from his lecture delivered on presentation of the JAMES SCOTT prize, February 6, 1939[1] that "The physicist, in his study of natural phenomena, has two methods of making progress: (1) the method of experiment and observation, and (2) the method of mathematical reasoning. The former is just the collection of selected data; the latter enables one to infer results about experiments that have not been performed. There is no logical reason why the second method should be possible at all, but one has found in practice that it does work and meets with reasonable success. This must be ascribed to some mathematical quality in Nature, a quality which the casual observer of Nature would not suspect, but which nevertheless plays an important role in Nature's scheme."

To follow Dirac further, he lays prime importance to mathematical beauty in physical theories. It is not simplicity but mathematical magnificence which a modern physicist covets when he is searching for a new theory. This implies in a most forceful manner that it is mathematical beauty over anything else which gives a onward thrust and guiding direction to a developing and fledgling theory of physics. A fresh way of finding a new physical theory of fundamental interest is then to take a reverse path and start with a mathematical branch that has the possibility of becoming the basis of a new theory.

The physical theories apply to the real observable world fraught with all its complexities. The description of the world is segregated into two distinct parts,(1) the fundamental part which describes only the nature of interactions and the behavior of the natural world under naive conditions in the form of the physical laws and (2) the complicated part which draws on the complexities of the physical world and is termed as the initial conditions prevailing. These initial conditions are the given part of a physical problem with no reason attached to their existence. The modern mathematical physicist if we may call him so dreams of arriving at these initial conditions as well on the basis of some as yet undiscovered mathematical structure. It is ingrained in our thinking that maths and theoretical physics are a two way process pointing at the same end viz: description of an abstract phenomenon. It is mathematics that determines the content and meaning of physical concepts and theories themselves[2]. Physical theories and the appropriate mathematical framework evolve in parallel, often as the result of the work of the same persons. This is the case of the foundations of infinitesimal calculus and of classical mechanics in the 17th century, mainly through the work of Newton and Leibniz; or, the parallel development of vector analysis and of electromagnetic theory in the second half of the 19th century, mainly by Maxwell, Gibbs and Heaviside.

Einstein's General theory of relativity also developed with a matching development in tensor analysis.
Thus any description of a mathematical idea is incomplete without adequate description of the historical development of the concerned theory and also its interrelation with other sciences. The same applies for physics education.

From an epistemological point of view, mathematics and physics are closely related. Both rely heavily on logic. The very nature or the root of the problems dealt with in both the sciences is the same and is driven by similar guiding force. Both provide for the other a grazing land to feed on while formulating new theories and each new theory becomes a framework for the other to stand on. The content of each is largely fashioned by the other and meaning of the content cannot be ascribed to any one of them alone.

II. TWO DIFFERENT PROOFS OF PYTHAGORAS THEOREM

The Pythagoras theorem is one of the most ancient geometrical theorem which relates three sides of a right triangle. It states that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. The theorem has numerous proofs, possibly the most of any mathematical theorem. These are very diverse, including both geometric proofs and algebraic proofs, with some dating back thousands of years. Here I compare two proofs, one of which is developed by the author and the other standard geometrical proof found in text books[3]. The thinking of a mathematical mind is marked out from a conventional thinker of physics by this comparison.

$$\frac{AC}{AB} = \frac{AB}{AD}$$

The geometrical theorem is given below: The proof is based upon the proportionality of sides of two similar triangles. In Euclidean geometry the ratio of two corresponding sides of two similar triangles is the same for all the three pairs of corresponding sides of the two similar triangles. Now refer to fig. 1.

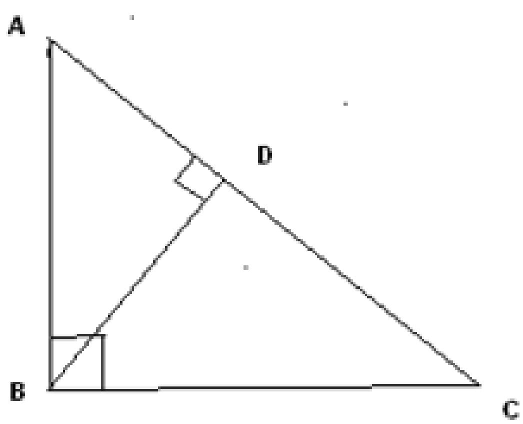

Figure. 1.

The triangle ABC is a right triangle. A perpendicular BD is drawn on AC from point B. Triangle ABD is similar to triangle ABC as two angles (common angle A and two right angles, angle ABC and angle ADB) are equal. The corresponding sides are (AC,AB), (BC,BD) and (AB,AD) where first entry is for ΔABC and the second is for Δ ABD. Similarly triangle ABC and triangle BDC are also similar and therefore the corresponding sides are: (AB,BD),(BC,DC) and (AC,BC). The ratio of the corresponding sides should be equal so:

$$\frac{AC}{AB} = \frac{AB}{AD} \quad \text{and} \quad \frac{BC}{DC} = \frac{AC}{BC} \qquad (1)$$

which gives : $AB^2 = AC \times AD$ and $BC^2 = AC \times DC$.

This gives on addition of the two equations:

$$AB^2 + BC^2 = AC \times AD + AC \times DC = AC \times (AD + DC) = AC^2$$

which proves the Pythagoras theorem:

$$AB^2 + BC^2 = AC^2$$

Now the same theorem is proved by a different method. Consider figure 2.

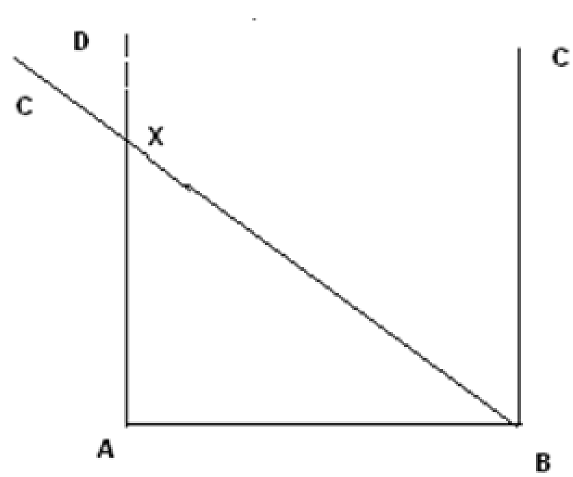

Figure. 2.

In the above figure AD and BC tends to infinity. Thus both the rays meet at infinity and form a triangle with two right angles and one angle of 0^0. Let us assume that Pythagoras theorem applies to this triangle. This should be the case in the given condition as the two sides tend to infinity. Thus:

$$AD^2 + AB^2 = BC^2 \qquad (2)$$

Now we vary the angle ABC such that BC cuts AD at X. Triangle ABX is a right triangle.

We need to prove that :

$$AX^2 + AB^2 = BX^2 \qquad (3)$$

which in turn means :

$$(AD - XD)^2 + AB^2 = (BC - XC)^2$$

This gives on squaring:

$$AD^2 + XD^2 - 2AD \times XD + AB^2 = BC^2 + XC^2 - 2BC \times XC$$

which on using (2) gives:

$$XD^2 - 2AD \times XD = XC^2 - 2BC \times XC$$

Or :

$$XD\,(XD - 2AD) = XC\,(XC - 2BC)$$

This gives on rearranging :

$$\frac{XD}{XC} = \frac{BC}{AD}\left(\frac{\frac{XC}{BC} - 2}{\frac{XD}{AD} - 2}\right)$$

Taking the limits BC andAD tending to infinity and considering that BC = AD we get

$$\frac{XD}{XC} = 1 \qquad (4)$$

As AD and BC tend to infinity, so does XD and XC. Since BC=AD ;

$$XD + AX = XC + BX$$

Or:

XD = XC + (a finite constant quantity).

Thus in the limit of XD and XC tending to infinity:

$$\lim_{XD, XC \to \infty} \frac{XD}{XC} = 1$$

which proves (4) , which in turn proves (3).

The following points can be noted from the above two proofs which are very different in nature:

1. A mathematical proof relies on a geometrical symmetry which provides conciseness and elegance to the proof. A proof which is physical in nature relies on a variational principle and depends on the boundary values of the problem.

2. The physical proof depends on the values of functions at variables tending to infinity or at large distances. This kind of thing is avoided in case of mathematical proofs. The governing principles in physics which are based on larger material picture (like conservation of mass or energy and momentum) are not there in mathematics. In maths and geometry the governing principles are mostly based upon mathematical beauty and geometrical elegance.

3. In the end the famous quote by another Nobel laureate in Physics, Richard Feynman, can sum up the difference between Physics and Maths, "It is not philosophy we are after, but the behavior of real things"[4].

III. CONCLUSION

The common grounds between Mathematics and Physics have been explored in this paper. Various commonalities and differences have been pointed out. Two different proofs of Pythagoras theorem have been given and basic differences between the two sciences have been highlighted based upon the proofs.

IV. ACKNOWLEDGEMENT

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Dr. Aniruddh Singh obtained PhD in Theoretical Nuclear Physics from Jamia Millia Islamia, New Delhi.He obtained BSc Hons in Physics from Delhi University and MSc Physics from IIT Kanpur. His PhD thesis is in the field of Variational Monte Carlo methods as applied to light nuclei and hypernuclei.

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